Necessary and Sufficient Conditions

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The discussion of 'necessary and sufficient conditions' is well understood in philosophy, and as a result, I sometimes make the mistake of assuming it is commonly understood in the wider community. This post redresses this by sketching the concept and why it is important.

Conditions

To say that one thing is a condition for another is to say that the one thing is involved in making the second thing happen.

The most common example of a condition is a cause. For example, striking a billiard ball with a cue causes the ball to move. Thus, the striking of the ball is a 'condition' of the movement of the ball.

But conditions need not be causes. Giving permission is another type of condition. For example, a driver's license gives you permission to drive. This, having a driver's license is a 'condition' for driving.

Necessary and Sufficient Conditions

There are two ways to express conditions:
B if A (alternatively: if A then B)
B only if A

The first is called a sufficient condition. The second is a necessary condition.

The idea of a sufficient condition is that it is enough to make something happen. For example, in most cases, pushing on the gas is enough to make the car go forward. It's not the only thing that would do it; you could make the car go forward by pushing it, for example.

The idea of a necessary condition is that something will not happen unless the condition happens. For example, we might say that the car will not go forward unless we have turned off the parking brake. Turning off the brake is thus a necessary condition to the car going forward.

Necessary and sufficient conditions are typically used to explain why something happens. "Why did the car go forward?" we ask. The brake was turned off; that was necessary for the motion to happen. And then somebody pressed on the gas; that was sufficient to make it move forward.

The Logic of Conditions

The logical structures of necessary and sufficient conditions do a dance around each other.

The simplest statement of a sufficient condition is as follows:

If A then B

This is equivalent to:

If not B then not A, and
It is not equivalent to:

If B then A

Meanwhile, the simplest statement of a necessary condition is as follows:

If B then A

And we often use special words to indicate this special status:

B only if A

Not B unless A

This is also equivalent to:

If not A then not B

And it is not equivalent to:

If not B then not A

**The Conditional Fallacy**

Why is this important? Because it points to what is probably the most common fallacy involving conditions: *not sufficient means not necessary.*

For example, we often hear this kind of argument:

Studies show that simply spending money will not improve test scores in schools. So we should be looking at something else, like quality teachers.
What makes this a bit tricky is that the conclusion is often implicit. The conclusion, if spelled out, is that we should be doing something instead of throwing money at the problem.

Here's an example of the fallacy being committed. Ewan McIntosh writes, "In 2006 there was $2 trillion spent on education by the world's governments. But money alone is not the reason we see improvement, not always." He then recommends "Getting the right people to become teachers, developing them into effective instructors (and) ensuring that the system is able to offer the best possible instruction for every child ." Presumably, instead of spending money on the problem - after all, Singapore didn't have to.

Here is Tom Hoffman identifying the fallacy in McIntosh's reasoning: "I don't have the slightest idea what school budgets look like in Scotland, so maybe over there it is appropriate to put across the message that more funding isn't necessary to improve education, but on this side of the pond, even this study makes it clear that improving American education requires spending more money."

The situation is represented thus: spending money is necessary but not sufficient to improve educational outcomes.

What this means is that simply spending money won't solve the problem. There are many ways to spend money that are not effective, as evidenced by many actual spendings of money that are not effective. Purchasing each mathematics class a Lear jet, for example, would certainly spend money. But it would not be very effective.

The response to this fallacy is to say, as Tom Hoffman did, that spending money is necessary in order to solve the problem. What this means is that, while the mere spending of money is no guarantee, nonetheless, the problem will not be solved unless money is spent. The supposition that the problem can be solved without spending
money is a fallacy.

Causation

As you may imagine, with the logic of conditions being so entwined, it is very easy to get tangled in a mess of necessary and sufficient conditions. This is especially the case when attempting to state whether one thing will cause another to happen.

Many people mistake a cause as the sufficient condition for something to happen (sometimes thought of as the 'efficient cause [https://edtechbooks.org/-CzX]' or the 'causal agent'). But formally, we should think of a 'cause' as 'a necessary and sufficient condition for an effect'.

That is to say, the description of a cause needs to include, not only the sufficient conditions, but also the necessary conditions, for an effect.

So if we sat that 'A' is a set of necessary and sufficient conditions, then when we say that 'A causes B' we mean that: 'If A then B' and 'If not A then not B'

You need both parts to ascribe a cause. You need to show that when A happens, that B also happens, but also, that it is not a coincidence, that is, when A does not happen, B does not happen either.

Some people at this point may argue that only a correlation, and not a cause, has been established. They argue that, in addition to a correlation, a causal argument must also appeal to a general principle or law of nature. This may be the case; if so, then we can simply say here that showing that 'If A then B' and 'If not A then not B' is necessary, but not sufficient, to show that A causes B.
Ceteris Paribus

The phrase *ceteris paribus* is Latin for 'all other things being equal' and is an important principle for understanding the concept of necessary and sufficient conditions.

Strictly speaking, the description of a cause for any event would be endless. For example, if I wanted to say that 'the car caused the accident' then I would need to say that the car exists and that the accident happened and that the earth exists and that the laws of nature are as we understand and that the accident was not a sub-temporal sentient being and that Merlin did not intervene and... well, you get the idea.

Usually, when we say that one thing is a cause for another, or that one thing is a condition for another, we assume a certain background state of affairs, which continues as it always has. This is especially important when talking about sufficient conditions, but will also come into play when talking about necessary conditions.

When I said 'pressing on the gas was sufficient to move the car', I *assumed* that, as usual, the parking brake was not engaged. Because, after all, were the parking brake engaged, pressing on the gas might *not* be sufficient to move the car. Really, I should say, 'Releasing the parking brake and pressing on the gas is sufficient to move the car'. But since the parking brake is almost never engaged, it is not usually necessary to say this; I just assume it.

Similarly, when I said that 'releasing the parking brake is necessary to move the car', the presumption was that the parking brake was engaged. But most of the time, releasing the brake is *not necessary* because the brake was not engaged in the first place. I do not need to state the necessary condition.

This is why the concept of 'control' is so important in scientific
experimentation. If you say 'all else being equal', then if you are measuring for results, then you need to know that all else was, in fact, equal.

**Expectations**

When you say 'all else being equal', you are assuming that a certain state of affairs holds, described in shorthand as 'all else'. But, of course, *something* changes, for otherwise causation would be impossible.

When you say 'all else' you mean 'everything not affected by the cause'. But this is essentially a statement of *expectations*. When you say 'A caused B' what you mean, in full, is that 'A caused B *instead of C*', where C denotes the alternative that would have been the case, all else being equal, has A not occurred.

[Bas van Fraassen](https://edtechbooks.org/-yTY) explains this at length. When you plant sunflower seeds beside the house and they grow to be six feet tall, someone may ask, "Why did the sunflowers grow here?" What they *mean* is, 'what caused them to grow (instead of to not grow)?' and not 'what caused sunflowers to grow instead of rutabagas?'

When Tom Hoffman writes, sarcastically, "I don't get Ewan's Scottish spin on [this McKinsey (i.e., American) study](https://edtechbooks.org/-kyy) of educational systems around the globe," he is speaking of expectations. He is suggesting the production of a given effect involves spending *more* money in one context, where in the other the production for the same effect, it is implied, does not mean the spending of more money.
Tricks Involving Ceteris Paribus

This is where *ceteris paribus* gets tricky. Very often, the presumption 'all other things being equal' does not mean, strictly, *all other things*, but rather, a subset of other things, and specifically (and importantly), the *set of necessary conditions* for the effect to happen.

Let us suppose that McIntosh said: "We can hire better teachers, but we do not need to spend money in order to do so." This is a bit of a caricature, but it is implied in the suggestion that the problem will not be solved by spending money.

Strictly speaking, this is impossible. It is not possible to hire teachers without spending money. What can only be meant is that it is not necessary to spend *more* money. He is stating, in other words, that enough money is *already being spent* to hire quality teachers.

But, of course, this money is currently being spent on something else. So in this case, *ceteris paribus* means 'same amount of money spend' but does not mean 'spent on the same things'.

The unstated argument here is that the money being spent elsewhere should be reallocated to spending on quality teachers. But this very necessary condition remains unstated. This is a fallacy; the necessary condition is hidden in the *ceribus paribus* clause.

A similar fallacy exists elsewhere in the same argument. McIntosh writes,

Less than 1% of African and Middle Eastern children perform at or above the Singaporian average - to be expected, you might believe, because those Singaporeans must hemorrhage cash into their education system. Wrong. Singapore spends less on Primary education than 27 of the 30 OECD countries.
Fair enough. But is McIntosh recommending that finding for education in the UK be adjusted to match the funding provided to education in Singapore? Almost certainly not!

This is a case of *shifting ceterus paribus* clauses. In *Singapore*, 'all else being equal' means expenditures at Singapore's levels. But in Britain, this means something very different.

Why is this important?

Because, if the expenses in Britain are *not* the same as those in Singapore, this means that there is something *very different* about Singapore which makes it possible to spend much less on education. But if Singapore is very different in precisely this way, then it is a poor analogy and cannot be used to define 'all else being equal', for, in this case, 'all else' is very different.

**Summary**

Arguments involving the use of conditions and causation are often deceptive because of the misuse of necessary and sufficient conditions.

When reading such arguments, you should not be swayed into believing that something is not necessary simply because it is not sufficient.

You should also be wary of hidden, and often shifting, assumptions about necessary conditions implicit in (frequently unstated) *ceteris paribus* clauses.

When evaluating such arguments, ask yourself simple questions. Like: if they did A, would the result be B? If they did not do A, would B not result?
Trust your intuitions. And keep in mind that if the appeal, by analogy, is to something that is unfamiliar to you - like Singapore, or like Estonia - the reason is most likely to hide some hidden difference that makes them a special case.

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